B. Sc. (Sem. V) (CBCS) Examination

October – 2019

Statistics : S - 503

(Statistical Inference)

(New Course)

Faculty Code : 003

Subject Code : 1015044

Time : $2\frac{1}{2}$ Hours]   [Total Marks : 70

Instructions :

(1) All questions are compulsory.

(2) All questions carry equal marks.

(3) Students can use their own scientifice calculators.

1   (a)   Give the answers of following questions :   4

(1)   An estimator T_n which is most concentrated about parameter θ is the _____ estimator.

(2)   If an estimator T_n converges in probability to the parametric function $\tau(\theta)$, T_n is said to be a _____ estimator.

(3)   If the expected value of an estimator T_n is equal to the value of the parameter θ, T_n is said to be _____ estimator of θ.

(4)   The difference between the expected value of an estimator and the value of the corresponding parameters is known as _____.

(b) Write any one :   2

   (1) Obtain sufficient estimator of θ for $f(x, \theta) = \theta e^{-\theta x}$;
       where $0 \leq x \leq \infty$.

   (2) Obtain an unbiased estimator of θ for Poisson
       distribution.

(c) Write any one :   3

   (1) Obtain consistent estimator of μ for Normal
       distribution.

   (2) If $t'' = \dfrac{1}{2}(t + t')$ where t and t' is the most efficient
       estimator with variance v then prove that
       $Var(t'') = \dfrac{1}{2}v\left(1 + \sqrt{e}\right)$.

(d) Write any one :   5

   (1) If $x \sim N(\mu, \sigma^2)$ and μ is known then obtain unbiased
       estimator of σ.

   (2) If T_1 and T_2 be two unbiased estimator of θ with
       variance σ_1^2, σ_2^2 and correlation ρ, what is the best
       unbiased linear combination of T_1 and T_2 and what
       is the variance of such combination ?

2 (a) Give the answers of following questions :   4

   (1) For discrete variable Crammer-Rao inequality _____.

   (2) An estimator of $v_\theta(T_n)$ which attains lower bound for
       all θ is known as _____.

   (3) An unbiased and complete statistic is a _____
       estimator provided MVUE exists.

   (4) If T_1 and T_2 are two MVU estimator for $T(\theta)$,
       then _____.

(b) Write any one : 2

   (1) Define Minimum Variance Bound Estimator.

   (2) Obtain MVUE of parameter p for Binomial distribution.

(c) Write any one : 3

   (1) Obtain MVUE and MVBE of θ for Poisson distribution.

   (2) Independent observation x_1, x_2, x_3, ..., x_n taken from the following density function

$$f(x, \theta) = \frac{1}{\theta} e^{-\left(\frac{x}{\theta}\right)}; \text{ where } 0 \le x \le \infty.$$

   Find the Cramer Rao Lower Bound for variance of unbiased estimator of θ.

(d) Write any one : 5

   (1) State and prove Cramer-Rao Inequality.

   (2) Obtain MVUE and MVBE of σ^2 for Normal distribution.

3 (a) Give the answers of following questions : 4

   (1) A value of a parameter θ which maximize the likelihood function is known as _____ estimate θ.

   (2) Maximum likelihood estimate of the parameter θ of the distribution $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$ is _____.

   (3) For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is _____.

   (4) Minimum Chi-square estimators are not necessarily _____.

(b) Write any one : 2

    (1) Obtain likelihood function of Negative Binomial distribution.

    (2) Estimate parameter θ by the method of moment for the following distribution

$$f(x, \theta) = \frac{1}{\theta} x; \text{ where } 0 \le x \le 1.$$

(c) Write any one : 3

    (1) Obtain MLE of parameter p for the distribution

$$f(x, p) = pq^x; \text{ where } x = 0, 1, 2, \ldots, \infty.$$

    (2) Estimate parameters α and β by the method of moment for the following distribution

$$f(x, \alpha, \beta) = \frac{\alpha^\beta}{\overline{|\beta}} e^{-\alpha x} x^{\beta-1}; \text{ where } x \ge 0, \alpha > 0.$$

(d) Write any one : 5

    (1) Obtain MLE of α and λ for the following distribution

$$f(x, \alpha, \lambda) = \frac{1}{|\overline{\lambda}} \left(\frac{\lambda}{\alpha}\right)^\lambda e^{-\left(\frac{\lambda}{\alpha}\right)x} x^{\lambda-1}; \text{ where } 0 \le x \le \infty, \lambda > 0$$

where $\varphi(\lambda) = \frac{\partial}{\partial\lambda} \log|\overline{\lambda} = \log\lambda - \frac{1}{2\lambda}$ thus $\varphi'(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$.

    (2) For the double Poisson distribution

$$P(X = x) = \frac{1}{2} \frac{e^{-m_1} m_1^x}{x!} + \frac{1}{2} \frac{e^{-m_2} m_2^x}{x!}; \ 0, 1, 2, \ldots$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - \left(\mu_1'\right)^2}$.

4   (a)   Give the answers of following questions :   **4**

       (1)   A null hypothesis is rejected if the value of a test statistics lies in the _____ .

       (2)   Probability of _____ error is called level of significance.

       (3)   If β is the probability of type II error, the power of the test is _____ .

       (4)   Accepting H_0 when H_0 is false is _____ error.

   (b)   Write any one :   **2**

       (1)   Define MP test.

       (2)   Given a random sample $x_1, x_2, x_3, ..., x_n$ from the distribution with pdf

$$f(x, \theta) = \frac{1}{\theta}; \text{ where } 0 \le x \le \theta.$$

Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$; where $c = \{x : x \ge 0.8\}$.

   (c)   Write any one :   **3**

       (1)   Give a random sample $x_1, x_2, x_3, ..., x_n$ from distribution with p.d.f.

$$f(x; \theta) = \theta e^{-\theta x}; \ 0 \le x \le \infty, \ \theta > 0.$$

Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(2) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \dfrac{1}{2}$ against

$H_1 : p = \dfrac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error, type-II error and power of test.

(d) Write any one : 5

(1) State and prove the Neyman-Pearson Lemma.

(2) Use Neyman-Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in the case of Normal distribution $N\left(\theta, \sigma^2\right)$ where σ^2 is known.

5 (a) Give the answers of following questions : 4

(1) Likelihood Ratio test for testing a hypothesis, simple or composite, against a _____ or _____ alternative hypothesis.

(2) The decision criteria in SPRT depends on the function of _____ and _____ errors.

(3) To decide about H_0, SPRT involves _____ regions.

(4) Likelihood Ratio test is relation the maximum _____ estimates.

(b) Write any one : 2

(1) Define ASN function of SPRT.

(2) Define UMP test.

(c) Write any one :   **3**

   (1) Construct SPRT of Binomial distribution for testing

   $H_0 : \theta = p_0$ against $H_1 : \theta = p_1 \left(> p_0 \right)$.

   (2) Write the properties of Likelihood Ratio test.

(d) Write any one :   **5**

   (1) Let sample distribution $x_1, x_2, x_3, ..., x_n$ taken from

   $$f(x, \theta) = \frac{1}{\sqrt{2\pi}} e^{-\left\{ \frac{(x-\theta)^2}{2} \right\}} ; \text{ where } -\infty \le x \le \infty, -\infty \le \theta \le \infty.$$

   To likelihood Ratio $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

   (2) Give the SPRT for test $H_0 : \lambda = \lambda_0$ against

   $H_1 : \lambda = \lambda_1 \left(> \lambda_0 \right)$, in sampling from a Poission

   distribution. Also obtain its OC and ASN function.

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